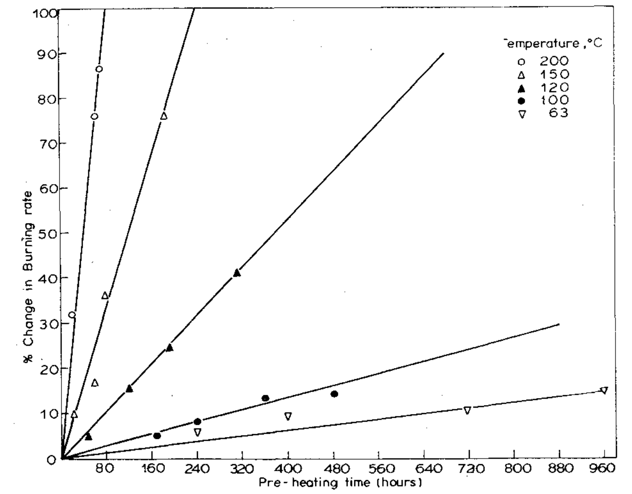

Fig. 1 Dependence of burning rate of PS/AP (75%) propellant at ambient pressure as a function of time and temperature of preheating.

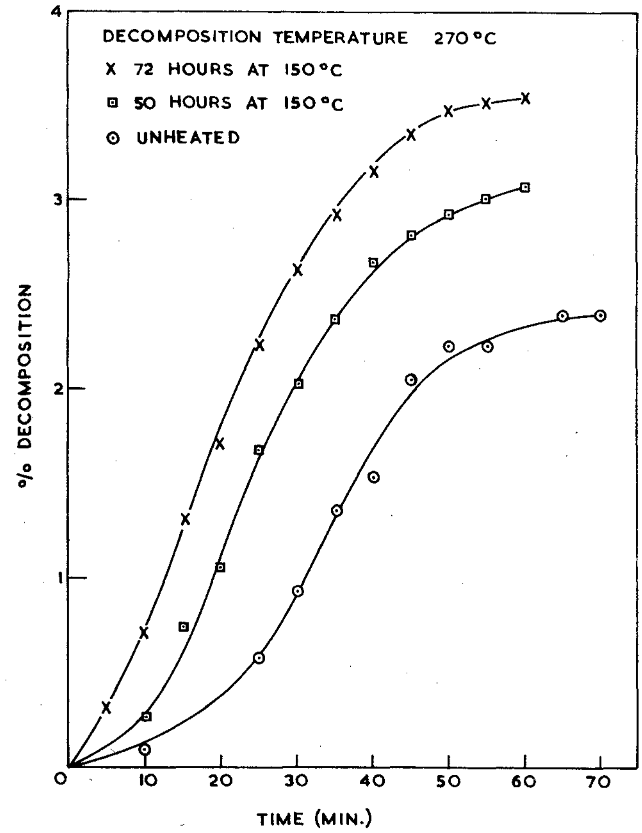

Fig. 2 Percentage decomposition vs time at 270°C of the preheated propellant at 150°C for different preheating time.

propellant shows a change in weight as a function of time and temperature of preheating. The data are presented in Table 1. It may be noted that the volume of the sample remains the same as in the preceding preheating experiments. This implies that the percentage weight loss is also the percentage change in density and that the gaseous products during preheating escape from the propellant and make the propellant porous. Therefore, one may argue that the observed change in the burning rate may be due to change in porosity of the propellant. Table 1 shows a comparison of the percentage change of the weight loss which in fact is proportional to the change in porosity and the percentage change in burning rate. It may be seen clearly that the percentage change in the weight loss, and hence the porosity, is almost negligible when compared to the extent of change of burning rate, thereby showing that the porosity has very little effect on the burning rate changes due to the preheating. However, even if the increase in the burning rate is due to the increase in porosity, the porosity itself is caused by decomposition and thus the change in the burning rate is related to the decomposition of the propellant. Further, we find that the propellant becomes yellow in color during preheating; this suggests that the increase in the burning rate is due to the accumulation of the yellow substance. Attempts are being made to analyze this yellow substance so that it could be used as a natural catalyst.

The present work, has shown that preheating of the propellant brings about a sensitization in the thermal decomposition and the burning rate of the propellant. It has shown further that a correlation exists between thermal decomposition and burning of the propellant. The present work together with earlier work[2] is a part of a program designed to clarify the role of condensed-phase reactions in combustion.

It may be of interest to note that Glaskova's data show pronounced desensitization in burning rate by additives known to inhibit the decomposition of AP.[5] This effect becomes insignificant beyond 200 atm, but in the pressure range of interest in rocket motors the data show a two- to threefold change. Similarly, Boggs et al. write that "The inclusion of dichromate into the crystal was responsible for increasing the deflagration rate (of AP) as a function of dopant concentration and pressure."[6]

To have a deeper insight into the role of decomposition during combustion, we have observed that the propellant in the actual combustion decomposes in a few layers below the surface. This residue has been subjected to analysis and our preliminary observations have shown that this residue is somewhat similar to the residue obtained in the partial decomposition of the propellant. Thus, the foregoing arguments indicate that the thermal decomposition of the propellant, and the AP could be related to the burning rate.

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On the Buckling of Shallow Spherical Caps Subjected to Uniform External Pressure

G. D. Galletly*
University of Liverpool, Liverpool, England

a = base radius of spherical shell
h = shell thickness
n = circumferential wave number

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*Professor of Applied Mechanics.

p_{cr} = buckling pressure, experimental or theoretical
p_{cl} = linear classical buckling pressure $=2[3(1-\nu^2)]^{-1/2}E(h/R)^2$
q_{cr} $=p_{cr}/p_{cl}$ = nondimensional buckling load
w = radial deflection
E = Young's modulus
H = rise of spherical shell
R = radius of spherical shell
α = angle measured from axis of revolution to edge of spherical shell
ϵ = strain
λ = geometric parameter, see Eq. (1)
υ = Poisson's ratio
σ_{yp} = yield-point stress
A = cross-sectional area of ring
A^* $=A(R/h)^{1/2}/(Rh)$

Introduction

THE buckling of spherical caps subjected to uniform external pressure (see Fig. 1) is a topic which has engaged the attention of numerous investigators from both the experimental and theoretical points of view. A fairly recent review of the subject was given by Kaplan[1] in 1972 who indicated how the considerable gap between theory and experiment had narrowed over the years.

A paper on spherical cap buckling was also given in 1972 by Sunakawa[2] in which he presented many experimental results which had been obtained at the University of Tokyo. However, these results did not agree with generally-accepted theoretical predictions. A full description of the experiments, on carefully-manufactured PMMA (polymethyl metracylate i.e., perspex) spherical caps, may be found in Ref. 3. Many values of the parameter λ were investigated, where λ is given by

$$\lambda=2[3(1-\upsilon^2)]^{1/4}(H/h)^{1/2} \qquad (1)$$

Strain and deflection measurements were recorded in these tests in addition to the buckling pressures. The prebuckling radial deflections w were found to be small, with $w_{max}/h<1$ up to the occurrence of snapping. Also, 16 millimeter high-speed movies taken during the tests showed that the shell deformations before, during, and after buckling were axisymmetrical. Efforts were also made to ensure that the shell edges in the tests were rigidly clamped.

The experimental buckling pressures given in Refs. 2 and 3 are plotted herein as Fig. 2. Also plotted for comparison on Fig. 2 are the theoretical elastic nonlinear buckling predictions (see Ref. 1 for a discussion of these curves and an extensive bibliography). The solid line corresponds to symmetric buckling, the dashed line to asymmetric buckling. It can be seen from Fig. 2 that a large trough occurs in the theoretical buckling curve for $3<\lambda<6$, but that the Japanese experimental results do not agree with the theoretical predictions.

With regard to other recent experiments on the buckling of spherical caps having λ-values in the region $3<\lambda<5$, one may cite the investigation of Tillman[4] on PVC caps. In the main, Tillman felt his experimental results agreed with the theoretical symmetrical buckling curve of Fig. 2, although some of his plotted results were slightly above the predicted values. Other experiments with similar λ-values have been made, inter alia, by Krenzke and Kiernan[5] and their results are shown in Fig. 3. These models were small, were machined from an aluminum alloy billet, and had integral support rings.

The present Note describes the results of external pressure tests made by the author on some small metallic machined spherical caps having $\lambda=4.0$ and with integral edge rings. Two metals were used in the tests, viz. HY-130 steel ($E=20.7\times10^6$N/cm^2, $\sigma_{yp}\simeq93.1\times10^3$N/cm^2), and an aluminum alloy, Hiduminium 48† ($E=6.9\times10^6$N/cm^2, $\sigma_{yp}\simeq31.03\times10^3$N/cm^2). The geometric characteristics were (see Fig. 1)

$$\alpha=5.7°,\ R=254\text{ mm},\ 2a=50.44\text{ mm},\ h=0.51\text{ mm}$$

$$H/a=0.049,\ h/R=1/500,\ \lambda=4.0$$

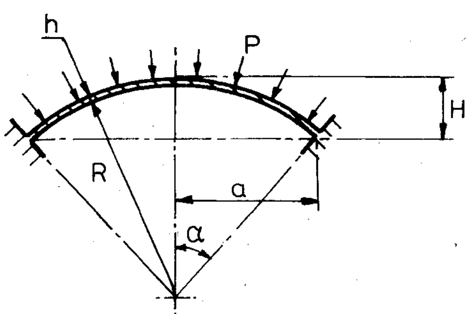

Fig. 1 Geometry of spherical cap.

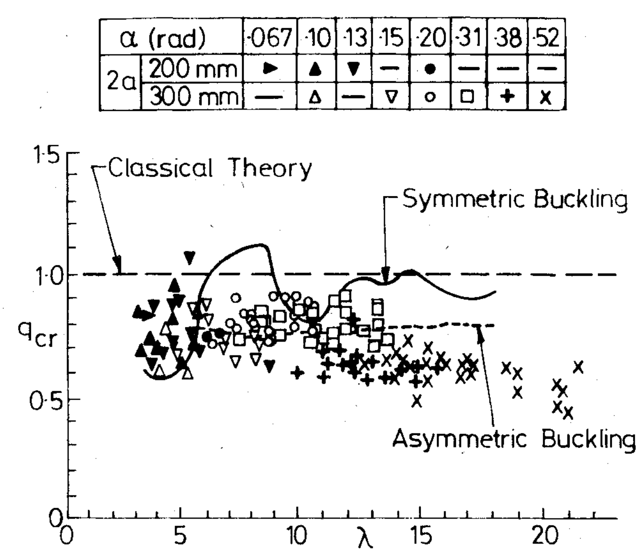

Fig. 2 A comparison of the experimental results of Sunakawa and Ichida with theoretical predictions.

Experimental Set-Up

The edges of the spherical caps in Refs. 2 and 3 were bonded to a supporting ring using an epoxy resin adhesive. For the present tests, however, it was decided to make the edge ring integral with the shell, and two types of ring were used (see Fig. 4 and Table 1). A tapered retaining ring was also employed in one of the tests (see Table 1) in an effort to ensure that the edge ring did not move horizontally outwards.

The HY-130 steel was in plate form, and the aluminum alloy in a cylindrical billet. In both cases, the specimens were rough-machined, stress-relieved, and then given a final machining operation (with the HY-130 steel, several rough-machining stages were used).

The external pressure was applied slowly and in stages using a hand pump. The snapping of the thin spherical shell at buckling was audible in all cases and the post-buckled shape for all the models was a symmetrical dimple.

Discussion of Results

The theoretical nonlinear elastic buckling pressures for the models, assumed to be perfect and to have clamped ends, are given in Table 1. They were found from the equation for p_{cl} in

†U. S. A. Alum. Assoc. No. 7039. The σ-ϵ curve of this material is a straight line over the range of interest.

Table 1 Results of external pressure tests on spherical caps

Model No.	Location of Ring-shell Junction	Buckling pressure N/cm^2 Experiment	Theory	Expt. p_{cr} / Theop_{cr}
HID-1	Top	22.7	19.6	1.16
HID-2	Top	23.6	19.6	1.20
HID-3	Mid-Height	20.9	19.6	1.065
HID-4	"	27.4[a]	19.6	1.40
HY-1	"	67.5	58.8	1.15
HY-2	"	53.8[b]	58.8	0.92

[a] This model also had a tapered retaining ring.
[b] This spherical cap was thin in parts.

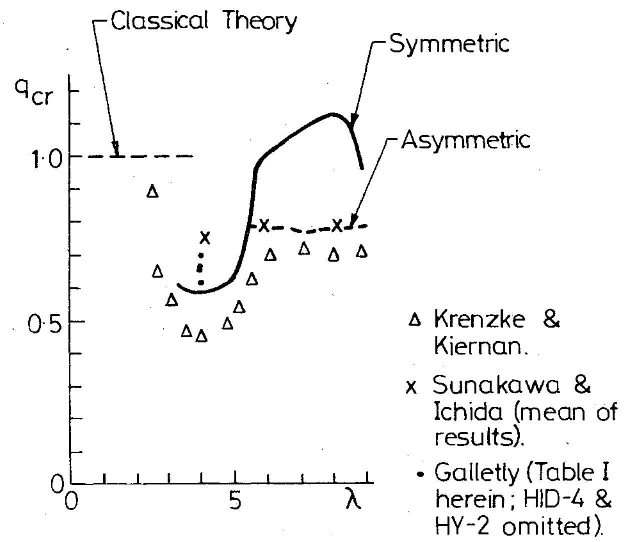

Fig. 3 Experimental and theoretical buckling pressures for spherical caps having low values of λ.

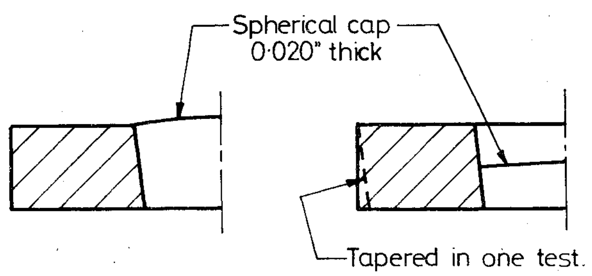

Fig. 4 Spherical cap and edge rings (schematic).

the Nomenclature, and the value of q_{cr} at $\lambda = 4.0$ in Fig. 2 (i.e., $q_{cr} = 0.58$).

The experimental pressures at which the models first buckled are also given in Table 1. The pressures at which the caps snapped back and the buckling pressures on reloading were recorded, but are not given herein.

As noted in Table 1, model HID-4 had a tapered retaining ring surrounding the edge ring. It is not clear whether the wedge action caused when this retaining ring was tightened up put a compressive ring load on the test assembly, and thus gave an augmented p_{cr}. The difference between the experimental p_{cr}'s for HID-3 and HID-4 (supposedly identical models) is certainly considerable. Again, model HY-2 was definitely too thin in parts, and its low experimental buckling pressure is understandable.

If the previous two models are left out of the discussion, one is still left with four models which had experimental buckling pressures greater than the theoretical. The percentage difference varied from +7% for HID-3 to +20% for HID-2 and, while these amounts are certainly lower than some of those obtained by Sunakawa and Ichida, they are all positive.

In evaluating the buckling pressures one has to consider the possible effects of boundary conditions, initial imperfections, (e.g. local radii of curvature, local thickness, pin-holes, etc.), residual stresses, and the evaluation of the material properties. As the present series of tests is a preliminary one, and only a few shells have been made and tested, it is difficult to make a proper assessment of foregoing points. However, the following may be said:

a) Boundary conditions: the size of the edge ring is fairly substantial in relation to the shell thickness (the parameter A^* used by Bushnell[6] is >150 for the present cases) and is held down to the base ring by 8½ in. diam. bolts. The assumption of clamped ends would thus appear to be a reasonable one.

b) Initial imperfections: apart from model HY-2, the models seemed to be reasonably free from geometric imperfections.

c) Material properties: the main property in the present context is the Young's modulus, E. For both materials, coupons were taken in the radial and circumferential directions and the results showed little anisotropy. However, the question remains as to how accurately tensile tests on cylindrical rods reflect the properties of thin sheets.

Bearing the previous factors in mind, it is quite possible that the theoretical values in Table 1 may be too low and should be increased. However, at the present time, there is not sufficient information whereby this can be done properly.

Conclusions

External pressure tests on a small number of machined metallic spherical caps gave experimental buckling pressures which were slightly higher than those predicted theoretically, but which were not as high as those obtained by Sunakawa and Ichida. More work on the subject would appear to be desirable in order to duplicate and/or account for the high Japanese results.

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